NON-STEADY-STATE HEAT EXCHANGE OF LINEAR UNDERGROUND CONSTRUCTIONS WITH A THAWING (FREEZING) SOIL

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An approximate analytical method is presented for calculation of the temperature distribution of a heating agent over the length of underground channels of arbitrary form for arbitrary depth in thawing (freezing) surrounding soil in a non-steady-state heat-exchange regime.

When linear engineering constructions are located in a mass of soil which thaws or freezes because of their thermal effect, it is necessary to know the thermal regime of the construction and the position of the boundary between frozen and thawed zones of the soil at a given time. A general method for calculation of thaw (freeze) zone formation of soils for type I and III boundary conditions on the soil-construction boundary together with the total thermal flux through this surface was presented in [1] for a known thermal regime of a buried volume construction. On the basis of this method a quite simple closed solution was obtained in [2] for the case of steady-state heat exchange of isolated and paralleled linear constructions (tubes, ventilation channels, mines, etc.) with thawing (freezing) soil with the following simplifying assumptions [3-5].

1. At the initial time the entire soil mass is frozen (thawed) and has an identical temperature T_0.

2. In the initial section of the construction (at $x = 0$) the temperature of the heat-exchange agent is constant over time ($\theta(0) = \theta_0 = \text{const}$), on the surface of the soil (at $z = 0$) the temperature T_0 is maintained, as a result of which there is formed about the construction a zone of thawing (freezing) soil which varies with time and distance along the construction axis.

3. Ice-water phase transitions in the soil occur abruptly at the boundary of the thawed and frozen zones at a temperature $T_f = 0°C$, with the thermophysical characteristics of the soil also changing abruptly.

4. Heat transport within the soil occurs solely due to thermal conductivity under the action of the difference between temperatures T_0 and $\theta(x, t)$, with heat transport due to the geothermal gradient being negligible.

5. The projections of the thermal flux vectors in the soil on the longitudinal axis of the construction are negligibly small, so that the temperature field in the frozen and thawed soil zones is defined by the solution of the Laplace equation for a two-dimensional region of the configuration under consideration, perpendicular to this axis.

6. The conductive component of heat transport along the longitudinal axis of the construction is negligibly small in comparison to the convective component.

7. The temperature of the heat-transfer agent $\theta(x, t)$ and its velocity of motion u are averaged over the area of the construction cross section.

In the formulation used for the problem we do not consider the periodic component of heat exchange of the construction with the ground caused by seasonal variations of temperature on the top surface of the soil; and all calculations are performed relative to mean annual values of the parameters. The validity of such an approach increases with increase in the depth of the construction within the ground.

A non-steady-state solution of the problem in approximately the same formulation was obtained in [4] for the case of an isolated linear underground construction of circular cross section, the dimensions of which are much smaller than the distance to the top surface of the

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soil. In this particular case it is possible to reduce the thermal conductivity problem in the soil to an axisymmetric one.

Below we will present a more general solution of the problem formulated for the case of non-steady-state heat exchange of isolated and parallel-situated linear underground constructions of arbitrary cross section with thawing (freezing) soil with consideration of the thermal effect of the top surface.

With the assumptions made above, the change in mean-mass temperature of the heat-exchange agent along the axis of the construction at time t can be described by the following equation:

$$\rho C_p F u \frac{\partial \theta (x, t)}{\partial x} = -Q_s (x, t) + q, \tag{1}$$

where $Q_s(x, t)$ are specific heat losses of the construction to the soil (per lin. m).

As follows from solution of the corresponding Stefan problem for a semibounded soil mass with internal heat source of arbitrary form presented in [1], the thermal flux $Q_s(x, t)$ will equal

$$Q_s (x, t) = \frac{\lambda_t \varphi}{1 - f_p (x, t) + \psi} \theta (x, t), \tag{2}$$

where f (x, t) is the value of the soil-construction system configuration function on the surface of the boundary between thawed and frozen zones of the soil at the point with coordinate x at time t;

$$\varphi = - \int_s \frac{\partial f}{\partial n} \bigg|_s dS \text{ — system form-factor;} \qquad \psi = \frac{\lambda_t \varphi}{\alpha S}.$$

For a non-steady-state thermal regime the coordinate $z_p(0, t)$ of the boundary at the initial section of the construction is defined by the expression

$$I = \int_{Z_s}^{Z_p} \frac{(1 - f_p + \psi) f_p}{f_p - \beta (1 - f_p + \psi)} \frac{|f'_Z|}{[f'_Y]^2 + [f'_Z]^2} dZ, \tag{3}$$

where

$$f'_Y = \frac{\partial f}{\partial Y} \bigg|_p ; \quad f'_Z = \frac{\partial f}{\partial Z} \bigg|_p ;$$

$Y = y/l$, $Z = z/l$ are dimensionless coordinates; l is the characteristic linear dimension of the construction;

$$I = \frac{\lambda_t \theta_0 t}{q_p l^2}; \quad \beta = -\frac{\lambda_f T_0}{\lambda_t \theta_0}.$$

For constructions having a vertical plane of symmetry passing through their longitudinal axis, it is desirable to define $f_p(x, t)$ and $Q_s(x, t)$ at $Y_p = 0$, so that $f'_Y = 0$ and calculation of the coordinate $Z_p(0, t)$ with Eq. (3) is significantly simplified.

The solution of Eq. (1) upon substitution therein of Eq. (2) with the boundary condition $\theta(0, t) = \theta_0$ has the form

$$\theta (\bar{x}, t)/\theta_0 = \left\{1 + \bar{q} x_s (t) \int_0^{\bar{x}} \exp [a x_s (t)\, b (\bar{x}, t)]\, d\bar{x}\right\} \exp [- a x_s (t)\, b (\bar{x}, t)], \tag{4}$$

where $x_s(t)$ is the coordinate of the boundary between thawed and frozen soil zones on the surface of the soil-construction boundary:

$$\bar{x} = x/x_s (t); \; a = \lambda_t \varphi/(\rho C_p F u); \; \bar{q} = q/(\rho C_p F u \theta_0); \; b (\bar{x}, t) = \int_0^{\bar{x}} d\bar{x}/[1 - f_p (\bar{x}, t) + \psi].$$

The law of motion of the boundary $x_s(t)$ is found from Eq. (4) at $\bar{x} = 1$:

$$\bar{q}x_s(t) + \exp[-ax_s(t)\,b_0(t)] = \theta_{cr}/\theta_0, \tag{5}$$

where θ_{cr} is the temperature of the heat-transfer agent at $x = x_s(t)$;

$$b_0(t) = \int_0^1 d\bar{x}/[1 - f_p(\bar{x},\ t) + \psi].$$

At $\theta = \theta_{cr}$, $f_p = 1$, and the left side of Eq. (2) is equal to $Q_s = \lambda_m T_0 \varphi$ [1]. With consideration of this fact, the following relationship for determining the temperature θ_{cr} can be obtained:

$$\theta_{cr} = -\psi T_0 \lambda_f / \lambda_t \tag{6}$$

In those cases where the power of internal heat sources is significantly less than heat losses of the construction to the ground ($q \ll Q_s$), Eqs. (4), (5) simplify significantly, and the law of change of the coordinate $x_s(t)$ can be expressed by:

$$x_s(t) = \frac{1}{ab_0(t)} \ln(\theta_0/\theta_{cr}), \tag{7}$$

while the heat-transfer agent temperature distribution at $0 < \bar{x} < 1$ has the form

$$\theta(\bar{x},\ t)/\theta_0 = \exp[-ax_s(t)\,b(\bar{x},\ t)]. \tag{8}$$

To determine the form of the parameters $b_0(t)$ and $b(\bar{x},\ t)$ explicitly, we assume as in [4] that in the first approximation the following linear dependence is satisfied for the coordinate $Z_p(\bar{x},\ t)$:

$$\triangle Z_p(\bar{x},\ t) = \triangle Z_p(0,\ t)(1 - \bar{x}), \tag{9}$$

where $\triangle Z_p(\bar{x},\ t) = Z_p(\bar{x},\ t) - Z_s$; $\triangle Z_p(0,\ t) = Z_p(0,\ t) - Z_s$.

System (3)-(9) allows us to obtain in closed form a general solution of the problem of calculating formation of a thawed zone in long-term frozen soils surrounding a linear construction of arbitrary form, and also calculation of the non-steady-state temperature distribution of the heat-transfer agent along the axis of the construction within the limits of this zone.

When λ_t is replaced by λ_f and λ_f by λ_t in Eqs. (3)-(9) they become the solution of the corresponding problem of freezing of the surrounding soil by the construction (for placement of constructions with $\theta_0 < 0°C$ in moist soils at $T_0 > 0°C$).

It should be noted that in the formulation used for the problem (consideration of initial and boundary conditions by specifying an annual mean soil temperature T_0) the solution obtained does not extend to the initial period of construction use of the order of magnitude of 1 year [3], during the course of which it is necessary to consider the effect of the temperature distribution in the soil mass at the time the construction is placed in operation.

For practical calculations by the proposed method, the values of the form-factors and system configuration functions can be taken from data in [3, 6, 7] depending on the form of the construction, or determined by the technique presented in [2].

As an example, the proposed method was used to calculate the quantities $\Delta z_p(0,\ t)$, $x_s(t)$ and $\theta(x,\ t)$ for an isolated rectangular underground mine tunnel, ventilated by air at positive temperature with the following parameters: tunnel width $B = 6$ m, height $H = 4.5$ m, depth below surface of tunnel axis $h = 6.75$ m, $\lambda_f = 2.25$ W/(m·deg), $\lambda_c = 2$ W/(m·deg), $q_p = 6 \cdot 10^4$ kJ/m^3, $q = 0$, $\alpha = 1.46$ W/(m^2·deg), $T_0 = -5.3°C$, $\theta_0 = 6.0°C$, $u = 0.5$ m/sec.

The system configuration function f and its form-factor φ were determined by the electrothermal analogy method using planar models made of electrically conductive paper [2]. Results of these calculations as well as similar (same initial data) calculations for a series of parallel tunnels with interaxis distance of 8.1 m are shown in Tables 1 and 2.

To evaluate the accuracy of the proposed method, the results obtained with it were compared to a numberical solution of the corresponding problems (see tables). The problem

TABLE 1. Change with Time and Coordinate x of Parameters $\Delta z_p(0, t)$, $x_s(t)$, $\theta(x, t)$ for Isolated Rectangular Tunnel

Solution	Time yr	Δz_p, m	x_s, m	x, m: 200	400	600	800	856,2
Analytic	2	1,83	856,2	4,68	3,52	2,62	1,86	1,67
	10	3,75	987,3	4,92	3,93	3,08	2,33	2,14
Numerical	2	1,92	910,5	4,60	3,66	2,82	2,05	1,84
	10	4,11	1060	4,86	3,98	3,18	2,48	2,28

TABLE 2. Change with Time and Coordinate x of Parameters $\Delta z_p(0, t)$, $x_s(t)$, $\theta(x, t)$ for Series of Parallel Rectangular Tunnels

Solution	Time yr	Δz_p, m	x_s, m	x, m: 400	800	1200	1600	2000
Analytic	2	3,65	2160,2	3,84	2,46	1,58	1,02	0,59
	10	9,52	2227,9	3,96	2,60	1,66	1,10	0,66
Numerical	2	3,83	2296	3,77	2,56	1,71	1,12	0,68
	10	10,4	2392	3,91	2,63	1,80	1,18	0,74

formulated reduces to solution of the convective heat-transport equation in the channel

$$\frac{\partial\theta(x, t)}{\partial t} + u\frac{\partial\theta(x, t)}{\partial x} = -\frac{\alpha S}{\rho C_p F}[\theta(x, t) - T(x, t)|_s] \tag{10}$$

simultaneously with the nonlinear three-dimensional thermal conductivity equation in the soil

$$C_v(T)\frac{\partial T}{\partial t} = \nabla[\lambda(T)\nabla T], \tag{11}$$

for the numerical approximation of which the explicit finite difference two-layer method of [8] was used. The dependence of the effective volume heat capacity C_v and the thermal conductivity coefficient of the soil λ on temperature in Eq. (11) were expressed by the method of [9] in terms of the quantity of unfrozen water in the soil, taken as in [10] for the following initial data: $\gamma_{sk} = 1800\ \text{kg/m}^3$, $W_n = 0.2$, $W_f = 0.15$, $I_p = 0.07$, $C_f = 2420\ \text{kJ/(m}^3\cdot\text{deg)}$, $C_t = 3170\ \text{kJ/(m}^3\cdot\text{deg)}$.

Analysis of the results presented in Tables 1 and 2 indicates that the proposed analytical method has accuracy applicable for engineering calculations.

NOTATION

θ and T, temperatures of liquid and soil; $\theta(x, t)$, mean mass temperature of liquid in channel section with coordinate x at time t; θ_0, liquid temperature at x = 0; T_0, initial soil temperature; t, time; x, y, z, coordinates; l, characteristic linear dimension; S and F, perimeter and area of construction cross section; n, normal to the surface separating the media; ρ and C_p, density and specific heat of liquid; u, mean mass velocity of liquid; λ, thermal conductivity coefficient of soil; C_v, volume heat capacity of soil; q_p, heat of ice-water phase transition per unit volume of soil; W_n, total soil moisture; γ_{sk}, volume weight of soil skeleton; W_f, soil moisture at flattening boundary; I_p, soil plasticity number; Q, total thermal flux through surface; q, specific power of internal heat sources (per lin. m); α, heat exchange coefficient; f and φ, configuration function and system form-factor; $\psi=\lambda_t\varphi/(\alpha S)$, $\beta=-\lambda_f T_0/(\lambda_t\theta_0)$, $I=\lambda_t\theta_0 t/(q_p l^2)$, dimensionless parameters. Subscripts: t, f, values in thawed and frozen zones; p, on surface separating thawed and frozen phases; s, on surface separating soil construction.

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THE EFFECT OF THE CONFORMATION OF MACROMOLECULES ON THE HYDRODYNAMIC EFFECTIVENESS OF POLYACRYLAMIDE

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It was experimentally discovered that when the dimensions and the asymmetry of macromolecules are increased, their hydrodynamic effectiveness and stability improve.

The search for materials and the necessity of studying the molecular aspects of the phenomenon of reducing turbulent drag by additions of polymers are decisive for the interest in investigations of the effect of the conformation (dimensions and shape) and pliability of macromolecules on their hydrodynamic effectiveness and stability.

For most authors working in this field the objects of investigations were synthetic polyelectrolytes, viz., polyacrylic acid and hydrolyzed polyacrylamide (PAA); the conformation of their macromolecules can be easily changed by changing the hydrogen ion indicator pH of the solution [1-8] and by the quality of the solvent [9, 10]. However, the study of separate specimens of the mentioned polymers did not make it possible to trace the effect of the conformation on the hydrodynamic effectiveness of macromolecules with the same degree of polymerization upon transition from neutral to polyelectrolytic ones. Such an attempt was made in [11] but only in the region of neutral pH values.

Apart from some contradictory data in [1, 3], most authors note that the maximal effect in reducing the resistance is attained in the region of neutral and weakly alkaline pH values where the macromolecules of polyelectrolytes are in a bulking, extended state [2, 4, 5, 7, 8].

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